



University  
of Victoria

Graduate Studies

Notice of the Final Oral Examination  
for the Degree of Doctor of Philosophy

of

**CLAUDE ALLEN WADDLE**

BSc (Combined Physics and Biochemistry, University of Victoria, 2009)

BSc (Combined Computer Science and Mathematics, University of British Columbia, 2009)

**“Fast Spectral Multiplication for Real-Time Rendering”**

Department of Computer Science

Tuesday, April 17, 2018

1:00 P.M.

Clearihue Building

Room B007

Supervisory Committee:

Dr. George Tzanetakis, Department of Computer Science, University of Victoria (Supervisor)

Dr. Alexandra Branzan-Albu, Department of Computer Science, UVic (Member)

Dr. Wu-Sheng Lu, Department of Electrical and Computer Engineering, Uvic (Outside Member)

Dr. Michael Brill, Principal Color Scientist, Datacolor Inc. (Outside Member)

External Examiner:

Dr. Ulf Assarsson, Department of Computer Science and Engineering, Chalmers University of Technology, Sweden

Chair of Oral Examination:

Dr. Timothy Iles, Department of Pacific and Asian Studies, UVic

Dr. Stephen Evans, Acting Dean, Faculty of Graduate Studies

## Abstract

In computer graphics, the complex phenomenon of color appearance, involving the interaction of light, matter and the human visual system, is modeled by the multiplication of RGB triplets assigned to lights and materials. This efficient heuristic produces plausible images because the triplets assigned to materials usually function as color specifications. *To predict* color, spectral rendering is required, but the  $O(n)$  cost of computing reflections with  $n$ -dimensional point-sampled spectra is prohibitive for real-time rendering.

Typical spectra are well approximated by  $m$ -dimensional linear models, where  $m \ll n$ , but computing reflections with this representation requires  $O(m^2)$  matrix-vector multiplication. A method by Drew and Finlayson [JOSA A 20, 7 (2003), 1181-1193], reduces this cost to  $O(m)$  by “sharpening” an  $n \times m$  orthonormal basis with a linear transformation, so that the new basis vectors are approximately disjoint. If successful, this transformation allows approximated reflections to be computed as the products of coefficients of lights and materials. Finding the  $m \times m$  change of basis matrix requires solving  $m$  eigenvector problems, each needing a choice of wavelengths in which to sharpen the corresponding basis vector. These choices, however, are themselves an optimization problem left unaddressed by the method’s authors.

Instead, we pose a single problem, expressing the total approximation error incurred across all wavelengths as the sum of  $d \cdot m^2$  squares for some number  $d$ , where, depending on the inherent dimensionality of the rendered reflectance spectra,  $m \leq d \ll n$ , a number that is independent of the number of approximated reflections. This problem may be solved in real time, or nearly, using standard nonlinear optimization algorithms. Results using a variety of reflectance spectra and three standard illuminants yield errors at or close to the best lower bound attained by projection onto the leading  $m$  characteristic vectors of the approximated reflections. Measured as

CIEDE2000 color differences, a heuristic proxy for image difference, these errors can be made small enough to be likely imperceptible using values of  $4 \leq m \leq 9$ .

An examination of this problem reveals a hierarchy of simpler, more quickly solved subproblems whose solutions yield, in the typical case, increasingly inaccurate approximations. Analysis of this hierarchy explains why, in general, the lowest approximation error is not attained by simple spectral sharpening, the smallest of these subproblems, unless the spectral power distributions of all light sources in a scene are sufficiently close to constant functions. Using the methods described in this dissertation, spectra can be rendered in real time as the products of  $m$ -dimensional vectors of sharp basis coefficients at cost that is, in a typical application, a negligible fraction above the cost of RGB rendering.